Calculation Policy

Vision

HET schools deliver a broad and ambitious Maths programme of study which meets the requirements of the National Curriculum. Children are taught a coherent progression of skills and are given opportunities to answer a range of fluency, reasoning and problem solving questions. We teach content through a mastery approach following concrete, pictorial and abstract representations. Number sense and place value is vital for our learners to be efficient problem solvers who are able to reason and justify their thinking. Recalling basic number facts helps our children to think faster and more clearly, giving them the energy, attention and focus to tackle more complex questions.

We believe that maths is achievable for all and, so we teach through flexible groupings. Often, children work within mixed ability groups but at times more targeted challenge and support is beneficial and children may be grouped accordingly. We strive for every learner to feel motivated, empowered and capable so they are confident to apply their learning independently and in real life contexts. With this solid foundation, children have the skills and experience to enable them to develop a love for maths and the resilience to persevere when needed. Challenge to all is provided through deepening understanding rather than acceleration of content. By the end of their time at our schools, children are well equipped with a range of mathematical skills and strategies, which can be effectively transferred in different areas of the curriculum and prepare them for future successes.

Our children also have additional fluency lessons taught throughout the week in order to develop quick recall of basic number facts. In Key Stage 1, children have an additional 15 minute fluency session which is delivered using a Department of Education mastery programme. In Key Stage 2, children may revisit elements of this programme but also focus on times tables.

Fluency: the ability to recall and apply knowledge rapidly and accurately.

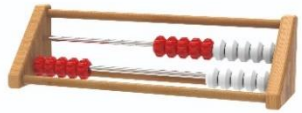
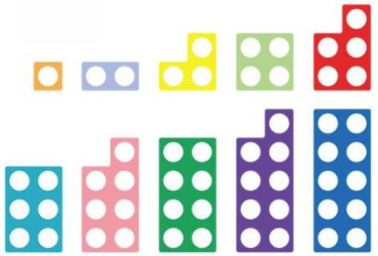
Reasoning: explaining their mathematical thinking

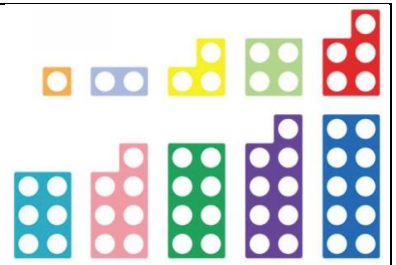
Problem solving: applying their knowledge to solve problems in varied contexts.

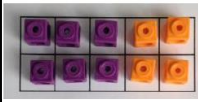
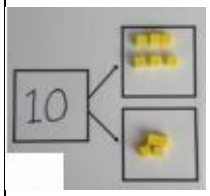
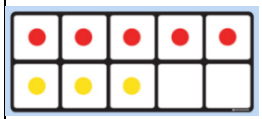
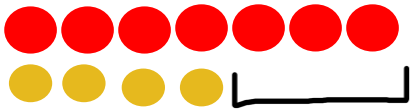
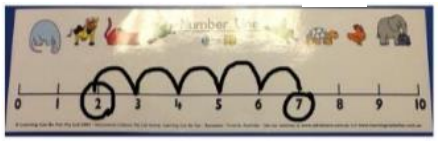
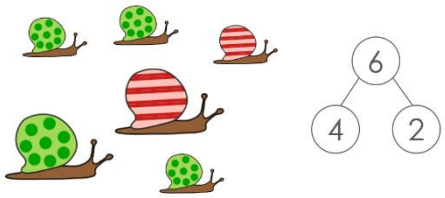
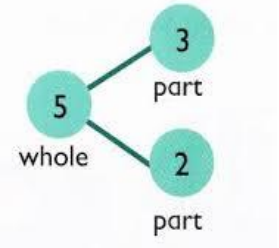
Maths is not always about 'big' numbers and times tables – it is about being able to apply concepts to different situations, problem solve, and find different strategies to check working.

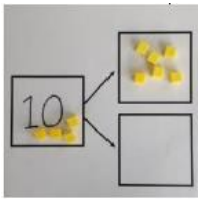
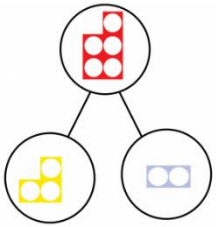
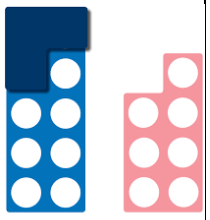
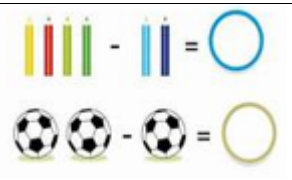
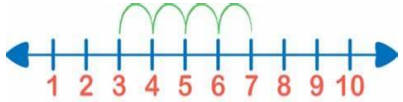
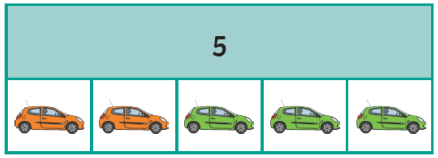
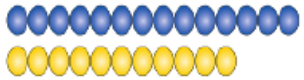
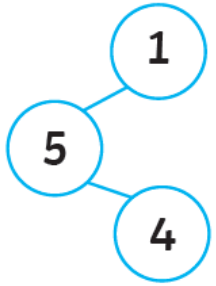
How to use this document

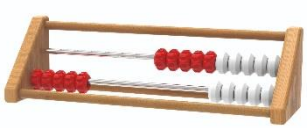
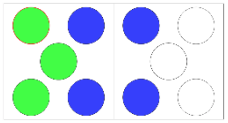
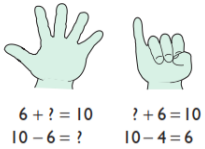
The document has been divided up into different year groups. The key objectives for the four operations (addition, subtraction, multiplication and division) and the methods within the concrete, pictorial and abstract stages to help see the progression of skills and knowledge throughout the skills. The video icon tell you that we have a video of the method being explained and demonstrated by one of our teachers.

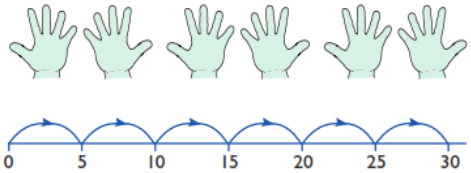
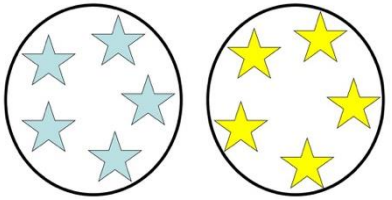
Reception Early Learning Goal	
Have a deep understanding of number to 10, including the composition of each number	Subitising - Children learn to subitise, which is the ability to recognise a small amount without needing to count. Children must be able to subitise an amount within 5, with some subitising to 10 by the end of EYFS. Counting – children understand that counting an amount matches with a number word – this is known as one to one correspondence. For example, children understand the word 5 could be 5 jumps, 5 cars etc... Cardinality – children learn that the last number said when counting is that in a set (giving a number to a set)  
Subitise (recognise quantities without counting) up to 5	 Numberblocks is used as a basis for number in EYFS. Episodes from series 1 and 2 are mostly used, as these link to the learning in class.
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including doubles facts.	Children use Numicon and Rekenrek counting frames to see the number bonds a number has. They can practise automatic recall in school and at home using the Numbots programme once they have learnt this objective.   
Verbally count beyond 20, recognising the pattern of the counting system	The main focus of this objective is for children to understand the pattern within the counting system. Children learn to understand the order of the ones and how this pattern starts again beyond 20, beyond 30 etc. Children learn this by counting during transition times but have many opportunities in the environment to consolidate this further, e.g. during self-registration.

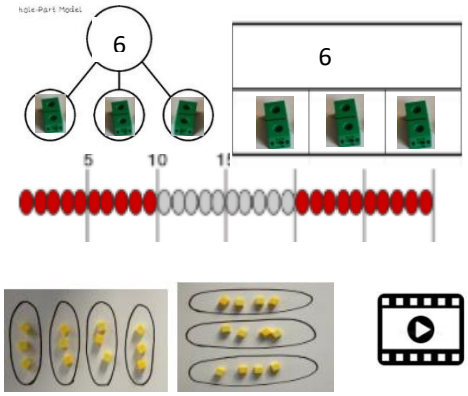
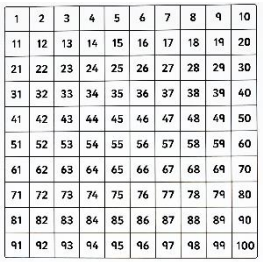
Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity	Children learn this concept by using resources to compare an amount. Numicon is a useful resource to use but also other objects are used when learning arises from play (who has more cars?). 
Explore and represent patterns within numbers up to 10, including evens and odds, doubles facts and how quantities can be distributed equally.	Children learn odds and evens using Numicon and Numberblocks (they can see the shape differs for odd numbers – we could see the odd tops and even tops). Children start to use the language of multiplication and division (sharing, equal groups etc) through play. 

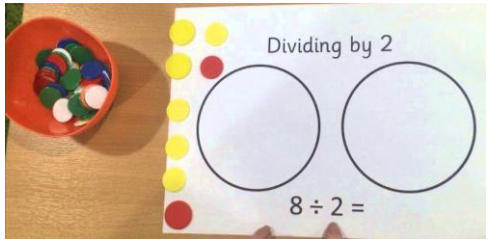
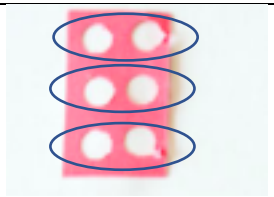
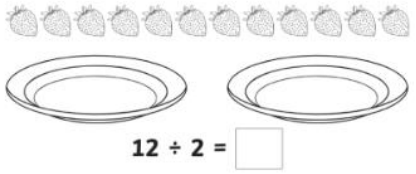
Year 1 – Addition			
Objective	**Concrete**	**Pictorial**	**Abstract**
Read, write and interpret mathematical statements involving addition (+) and equals (=) signs Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9 Add one-digit and two-digit numbers to 20, including 0	 Children use resources to physically put two groups together to see a total amount.    Children need to see addition in different variations e.g. in groups, using a tens frame, bar model or part whole grid. Children need to understand the parts add together to make the whole (total).  Emphasise that a full tens frame is always ten and if one row is complete, it is 5 and there is no need to count. From this, if there were 5 counts and a few more (5+3), children count on from 5 rather than from the beginning.   Children say STEM sentences to develop language and understanding e.g. 3 and 7 make 10. 10 is made of 3 and 7.	Once children are secure with using resources, they then look at pictorial representations.  3   It is also important to emphasise the language children may encounter with word problems when adding for example, first, then, altogether (first I had 2 pencils, then I got 4 more. altogether there were 6).	 When initially moving on to abstract representations, a part whole grid can bridge this gap as children can see how 3 numbers are used. As children become more confident, ask children to write calculations to show their understanding. 2 + 3 = 5 3 + 2 = 5 5 = 3 + 2 5 = 2 + 3

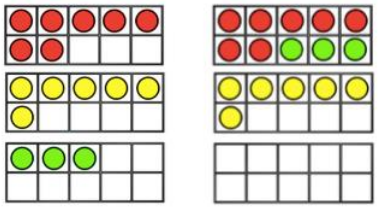
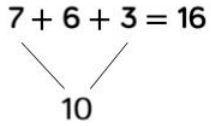
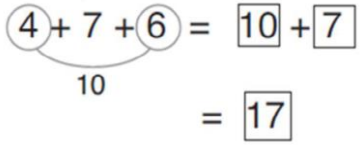
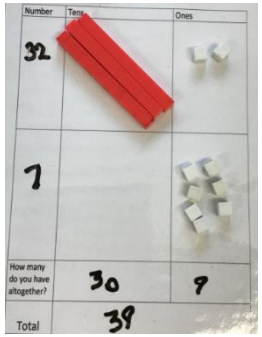
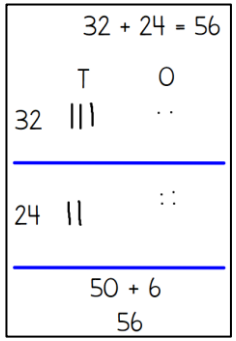
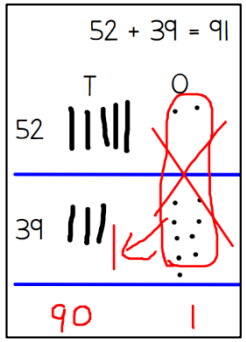
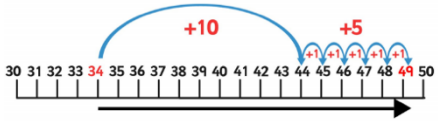
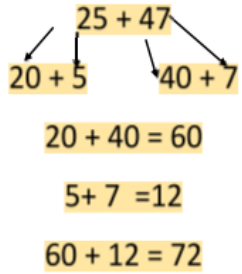
Year 1 – Subtraction			
Objective	Concrete	Pictorial	Abstract
Read, write and interpret mathematical statements involving subtraction (−) and equals (=) signs Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9 Subtract one-digit and two-digit numbers to 20, including 0	Children need to understand that when subtracting, they always start with the total (a larger number). From here, children move one resource to see the remaining number. Emphasise the language of take away / subtract. The part whole grid is a useful way of showing the relationship between addition and subtraction.   Children also place Numicon on top of each other with blacked out piece to show the order of subtract. e.g. first there is 10, then 3 is subtracted. Now there is 7.  10 – 3 = 7	 Once children are secure with concrete subtraction, pictorial representations should be encouraged.   A number line is a good bridge between pictorial and abstract representation. Encourage children to draw the jumps and emphasise counting as they jump to ensure they get to the correct number.  The difference between 11 and 14 is 3. Children need to also understand the subtraction can be seen as taking away but also finding the difference.	 When initially moving on to abstract representations, a part whole grid can bridge this gap as children can see how 3 numbers are used. As children become more confident, ask children to write calculations to show their understanding. 5 – 4 = 1 5 – 1 = 4 4 = 5 – 1 1 = 5 - 4

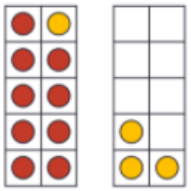
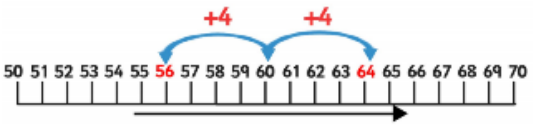
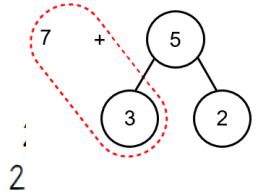
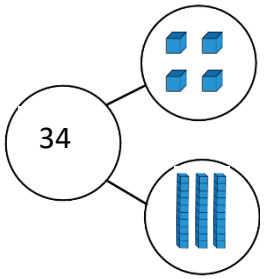
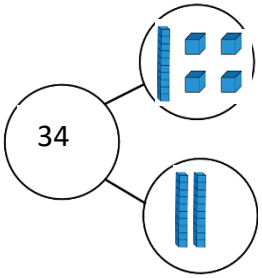
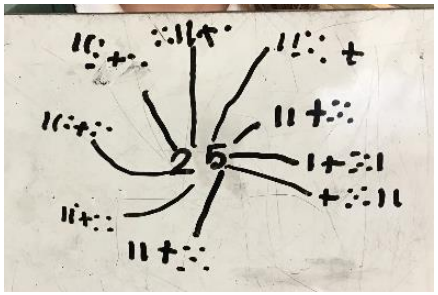
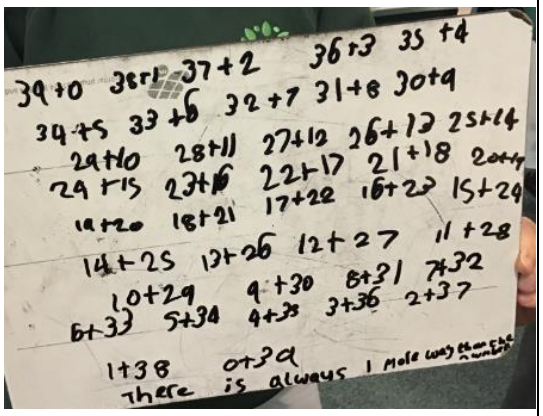
Combined objectives for addition and subtraction			
Represent and use number bonds and related subtraction facts within 20	 In school, children use different resources to learn number bonds within 20. Rekenrek counting frames (see above), particularly help children to see numbers in parts e.g. 6 can be seen as 5 'and a bit more' as Rekenrek's are in colour blocks of 5.	 The Hungarian number frame (dice) reinforces seeing number bonds within 10 in parts (e.g. 7 can be seen as 3 and 4).  6 + ? = 10 ? + 6 = 10 10 − 6 = ? 10 − 4 = 6	Children learn number bonds to 10 and 20 but should also be secure with bonds with 10. **(add link to website containing grid)** Children can practise automatic recall of these number facts in school and at home using the Numbots programme. 

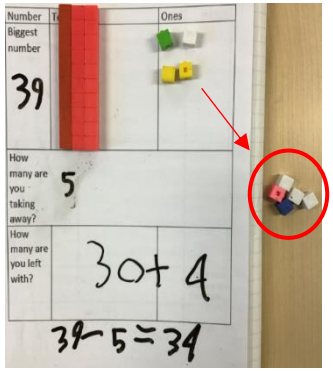
Year 1 – Multiplication			
Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	Children learn about repeated addition as a way of solving multiplication calculations. Children use objects to make equal groups and then add the groups together by skip counting in 2s, 5s and 10s. Doubles are also taught as part of multiplication. Children need to understand the difference between equal groups (same amount in every group) and unequal groups. 	 Children use skip counting to work out calculations. The relationship between adding an amount each time (repeated addition) and multiplying is needed so children truly understand that multiplying is an efficient way to add the same number together several times e.g. 5 + 5 + 5 + 5 + 5 + 5 can be seen as 5 multiplied by 6.	The main learning is around the language of multiplication in Year 1. Children may start to use the multiplication symbol alongside resources.  2 groups of 5 counters makes 10 counters altogether. The multiplication symbol is introduced to children who are very secure with the previous learning. They learn to read basic calculations

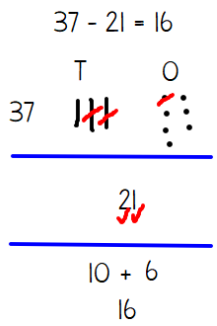
	Teachers show different representations of multiplication. The representations below show how multiplication is made up of repeated addition 	 When children learn to count in 2s, 5s and 10s, it is important that they use the 100 square to see the pattern of skip counting. Children are encouraged to use this alongside resources until they understand the patterns.	such as 5 x 2 = 10 but encouraged to show a concrete or pictorial representation alongside this to show their understanding.

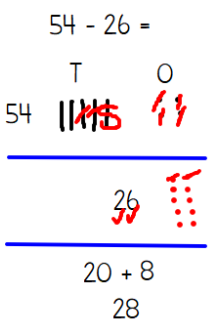
Year 1 - Division			
Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	 In Year 1, children practise division using resources. The language around this (sharing, equal groups).	 Although the main focus is using concrete objects, some children may start to see pictorial representations for division by circling part of the picture in order to represent the equal groups.	Children may start to learn how to write calculations for division but these are supported by concrete representations. 

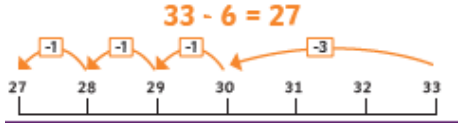
Year 2 - Addition			
Objective	Concrete	Pictorial	Abstract
Adding 3 1-digit numbers	  Use tens frames and different coloured counters to represent the 3-1 digit numbers	Children draw dots underneath each number or just Identify the number bond to 10 before drawing the remaining dots. 8 + 3 + 2 = 13 	 Combine the two numbers that make 10 and then add on the remainder.
Adding 2-digit to a 1-digit or 2-digit number	Children start by learning to add 2-digit + 1 digit numbers. Add together the ones first then add the tens. Use Dienes or if confident, use place value counters. Children do not cross the tens initially but then start to 'Swap Shop' (swapping tens ones for 1 lot of ten).   **Counting on** Using manipulative, make a number. Continue to add manipulatives as they count on.	**Pictorial column addition**   **Counting on** Children learn to use a number line (numbered or blank) to count on. First, counting by adding the tens and then the ones. 	**Partitioning**  Children will begin to use the partitioning method. Tens and ones will be added to form partial sums and then these partial sums will be added together to find the total. **Counting on** When confident children can start to mentally count on: • 2-digit + 1-digit by counting on in one • 2 – digit + tens (e.g. 35+40) by counting on in tens (using ten more knowledge) • 2-digit + 2 digit by counting on the number on tens followed by the amount of ones

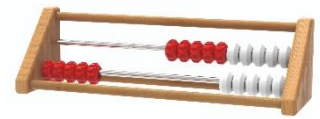
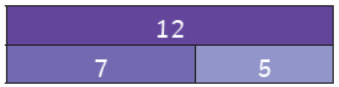
	‘Make 10 and then’ Children use a tens frame or other manipulatives to make ten and then add the remaining ones  9 + 4 = 13	**‘Make 10 and then’** When children are secure with making ten, this can be applied to 2-digit + 1 digit calculations e.g. 56+ 8 = 64 	**‘Make 10 and then’** Children do this method with pictorial support by partitioning the ones to make ten. e.g. 37 + 5 = 42 7 + 3 = 10 + 2 = 12 30 + 12 = 42  7 + 3 = 10 10 + 2 = 12
Partition any two-digit number into different combinations of tens and ones, explaining their thinking verbally, in pictures or using apparatus	Using Dienes, partition numbers into tens and ones   Children can also do this with other resources such as bundles of straws or place value counters. When children are confident to partition numbers into tens and ones and are able to write the correct calculation (30+4=34, 34-30=4), encourage children to partition the numbers in different ways by moving the tens, then ones. 	Partition numbers by drawing tens and ones. Gradually move tens over e.g. 20 + 5, 10+ 15. After this, start to partition the ones. Encourage children to write the calculations under each pictorial representation. 	Children then move to an abstract approach where drawing tens and ones isn’t necessary.  Encourage a systematic approach e.g. 30+0. 38+1. Children should start to make generalisations e.g. that there is always 1 more way than the number itself – there are 40 ways to make 39, 25 ways to make 24.

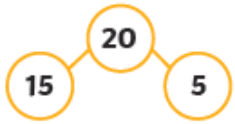
<table>
<tr><td colspan="4">Year 2 – Subtraction</td></tr>
<tr><td>Objective</td><td>Concrete</td><td>Pictorial</td><td>Abstract</td></tr>
<tr><td>Subtract 2-digit to a 1-digit or 2-digit number</td><td>Children start by learning to subtract 2-digit + 1 digit without crossing the tens. Use Dienes or if confident, use place value counters. Children then learn to do the same with 2-digit numbers (subtracting the ones then the tens). Before moving on to crossing tens where children need to ‘Swap Shop’ (swapping tens for ones)
 

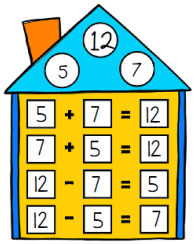
<u>Counting back</u>
Using manipulative, make a number. Subtract the manipulatives as they count back to find the answer.</td><td><u>Pictorial column subtraction</u>
 

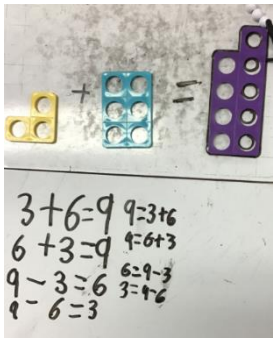
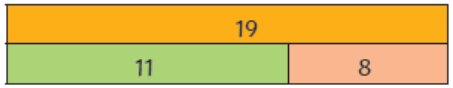
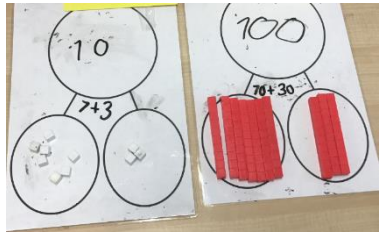
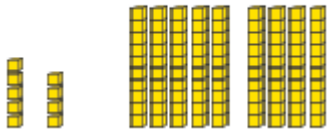
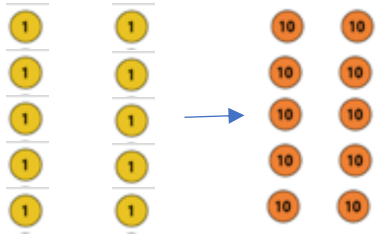
<u>Counting back</u>
Children learn to count back on a number, then blank number line. First counting back in jumps of tens then ones.
 
</td><td><u>Partitioning in order to count back</u>
Children partition the second number in a calculation in order to work out the answer
E.g. 84 – 22
84 – 20 = 64
64 – 2 = 62</td></tr>
<tr><td colspan="4">Combined objectives for addition and subtraction</td></tr>
<tr><td>Recall and use addition and subtraction facts to 20 fluently

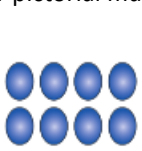
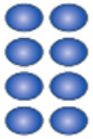
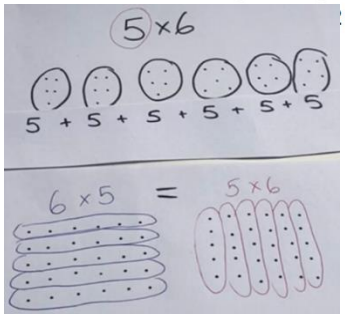
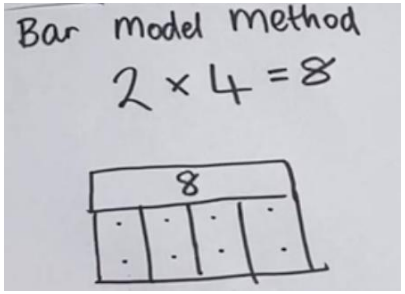
Recognise and use the inverse relationship between addition and subtraction and use this to</td><td>Children learn addition and subtract facts within 20 by making these numbers in different ways with manipulatives and writing the calulcations.
  </td><td>Children then move to a pictorial representation to write related facts.
 

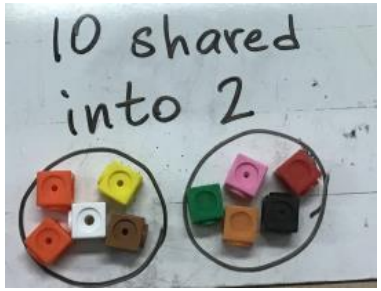
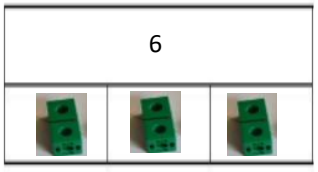
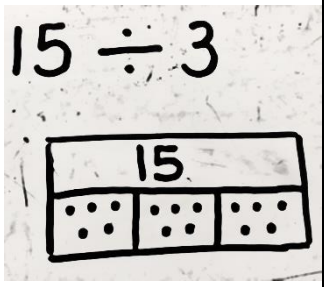
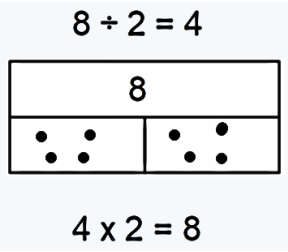
7 + 5 = 12
12 – 5 = 7
12 – 7 = 5
 

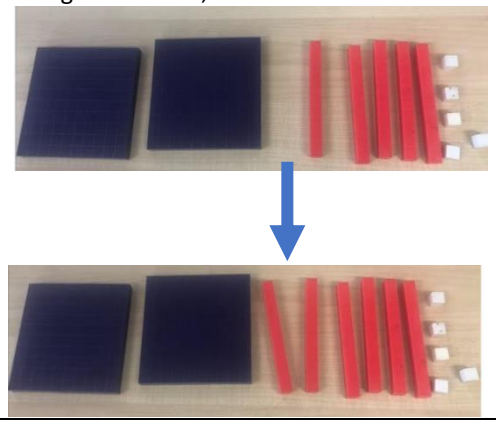
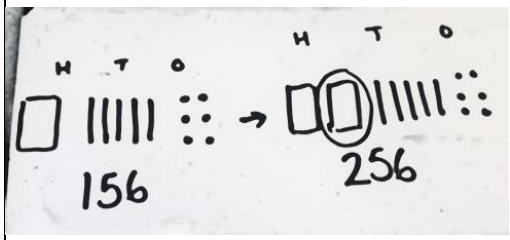
15 + 5 = 20
20 – 5 = 15
20 – 15 = 5
Children’s knowledge of related facts should support children in understanding that they can check calculations by doing the inverse.</td><td>Through secure understanding of addition and subtraction, children will be able to write related facts in an abstract format.
 

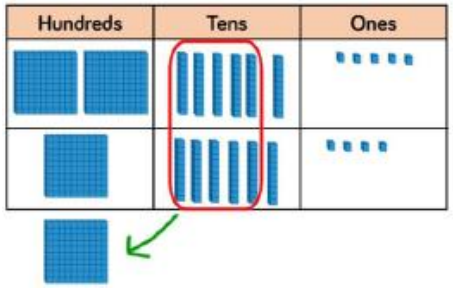
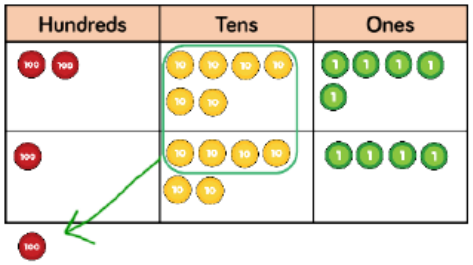
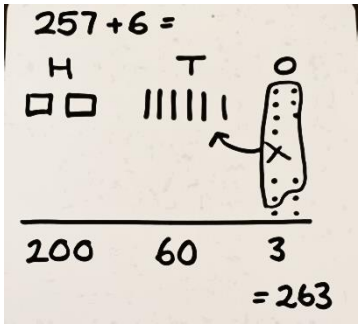
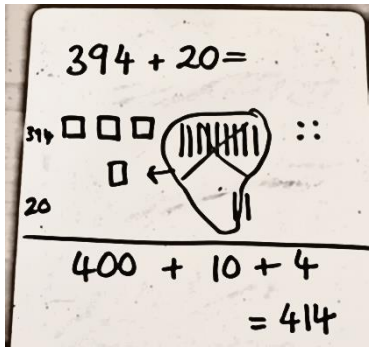
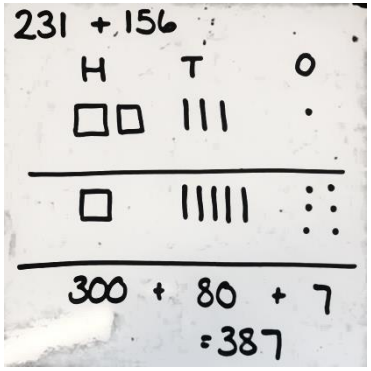
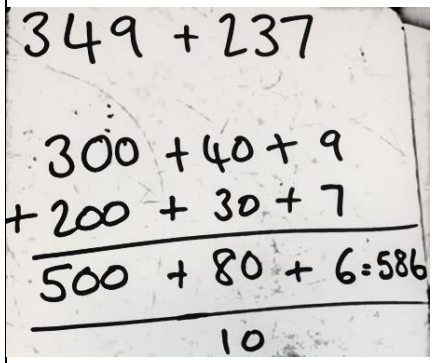
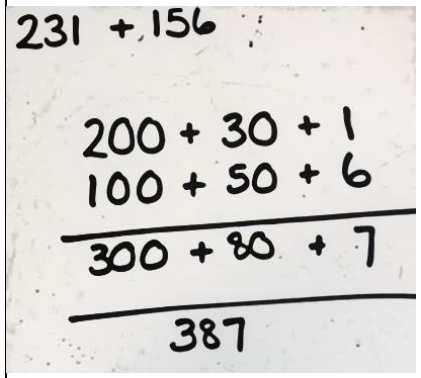
Children use addition and subtraction methods
(as described above) to check the inverse of a calculation.</td></tr>
</table>

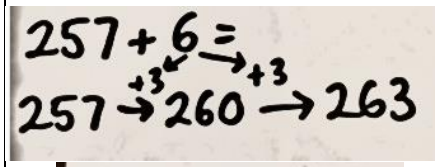
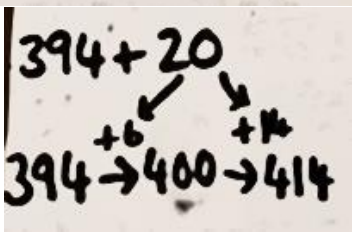
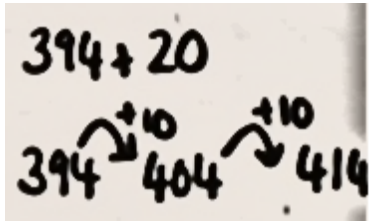
check calculations and solve missing number problems	 Children learn that subtraction is the inverse of addition. Children use resources to understand this and write the related facts about each calculation. Children are also encouraged to write calculation where the total is at the start. Children need to understand the = means 'the same as'.	 19 – 8 = 11 can be checked using 8 + 11 = 19	By the end of year 2, children should be able to quickly recall fluency facts within 20 (appendix 1)) **– add to year 1 too**
Derive and use related facts up to 100	 Children use their knowledge of number bonds to 10 in order to learn number bonds to 100. Manipulatives of Dienes or place counters work well	5 + 4 = 9 so 50 + 40 = 90  The same format is followed with pictorial representation. 	**<u>Compensating</u>** Children start to find more efficient methods when adding / subtracting including compensating. When calculations have 9 ones, children can quickly add/subtract 10 and then add/subtract the extra one **<u>Using fluency facts</u>** Children use fluency facts within 10 (including doubles and near doubles) and partitioning to support mental addition. 7 + 3 = 10 3 + 4 = 7 So 67 + 23 = 23 + 4 = 27 60 + 20 + 10 =

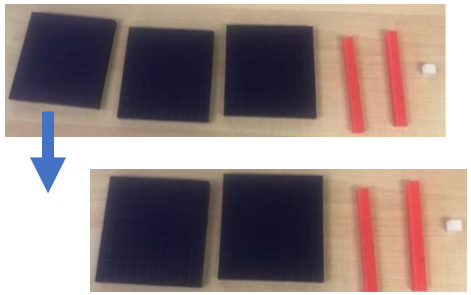
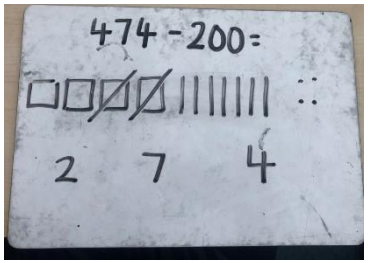
Year 2 – Multiplication			
Objective	**Concrete**	**Pictorial**	**Abstract**
Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×) and equals (=) signs Show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot Solve problems involving multiplication using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	Children make equal groups with resources. Children should place 1 object in each group at a time whilst counting.  The first number of a calculation is called the multiplicand. This is the number which will be multiplied. The second number is the number which we multiply. This is called the multiplier. Multiplicand x multiplier = total 2 x 4 = 8 This is the same as 2+2+2+2 = 8. Children learn that multiplication is commutative but learning the correct way to represent a calculation helps them when learning division.	Children learn to draw and read arrays as an early method for pictorial multiplication.  4 × 2 = 8 2 × 4 = 8  2 x 4 = 8 4 x 2 = 8  2 + 2 + 2 + 2 + 2 = 10  Children learn that multiplication is the same as repeated addition (adding equal groups). This helps them to grasp what the multiplication symbol means.  **Bar model** Children use the bar model to draw groups and then add the groups. They can do this by counting the dots or by counting in 5s.	Children then move on to abstract methods which firstly involve skip counting e.g. 5 x 3 (counting in 5s 3 times). Children should move towards being able to fluently recall 2s, 5s and 10s timetables. TT Rock stars is used to develop fluency in school and at home.

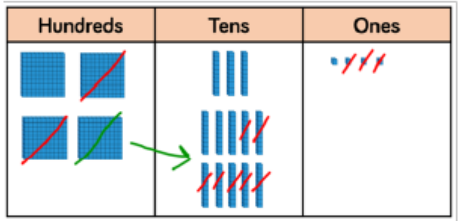
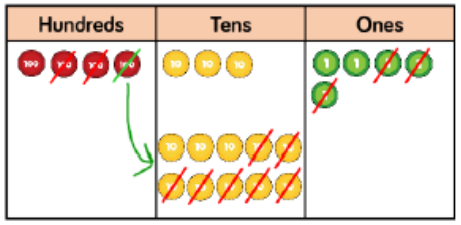
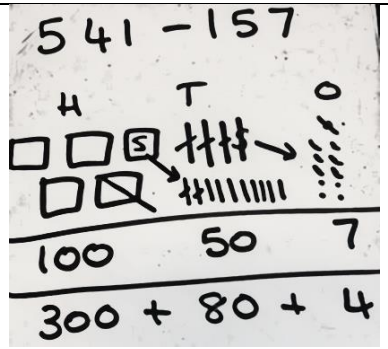
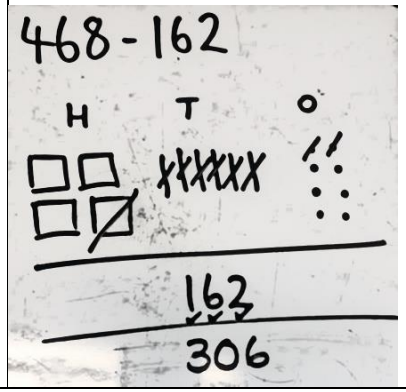
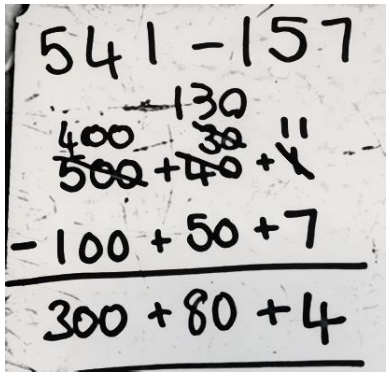
Year 2 – Division			
Objective	Concrete	Pictorial	Abstract
Recall and use division facts for the 2, 5 and 10 times tables, including recognising odd and even numbers Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the division (÷) and equals (=) sign Solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	Children use concrete resources to share amounts in equal groups. Emphasise to children that groups must be equal.  Children first practise sharing into equal groups with friends, hoops etc… before moving on to representing objects in the bar model. This will then help them to move on to the division method we use which is in the form of a bar model. 	The bar method is used as a pictorial representation for sharing. Children draw a bar model and share the dots between each group. This method helps children see the relationship between multiplication and division as the methods are similar. The same method is used for fractions as this is a form of division.  We encourage children to use the same representation for both multiplication and division (bar model) in order to see the relationship between the two operations. 	Children then move on to abstract methods which firstly involve skip counting e.g. 15 ÷ 5. Children would count in 5s until they get to 15 and note how many times they counted. Children should move towards being able to fluently recall division facts for 2s, 5s and 10s timetables. TT Rockstars is used to develop this fluency in school and at home.

Year 3 – Addition			
Objective	Concrete	Pictorial	Abstract
Add numbers mentally, including: • a three-digit number and 1s • a three-digit number and 10s • a three-digit number and 100s	In order for children to learn to mentally add 1s, 10s and 100s to a given number, children first start to make numbers with resources and add the given amount. This helps children to visually see how a number changes when a 1, 10 and 100 is added. 	To bridge the gap between making numbers with resources and mentally adding, drawing representations pictorially help to secure this place value knowledge. 	Children can use the representations from the concrete and pictorial learning to visualise numbers in their head.

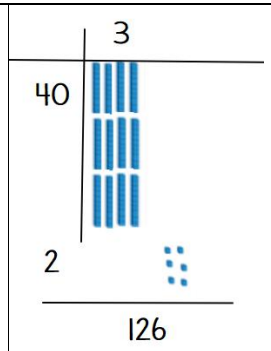
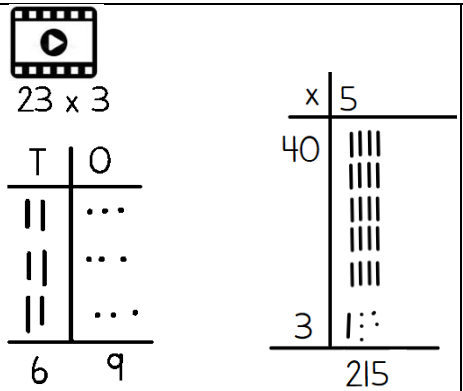
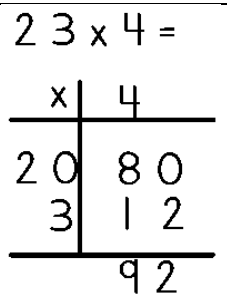
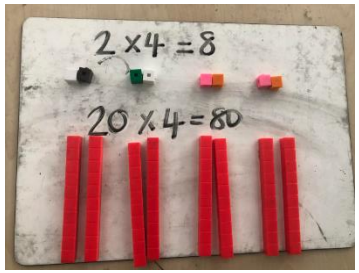
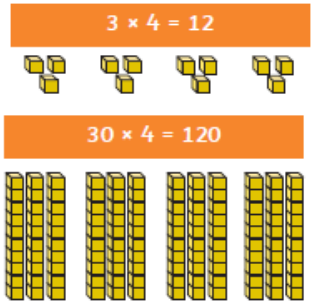
Add numbers with up to 3 digits, using formal written methods of columnar addition	Add together the ones first then add the tens. Use Dienes or if confident, use place value counters. Children do not cross the tens initially but then start to 'Swap Shop' (swapping tens ones for 1 lot of ten)   	   	200 + 30 + 5 + 70 + 4 300 + 00 + 9 = 309 100  

			<u>'Make 10 and then' (bridging)</u> Children can also use the 'make ten and then' method when adding. Partition    the second number in order to round up to the next multiple of 10 or 100 and then add on the remaining amount.

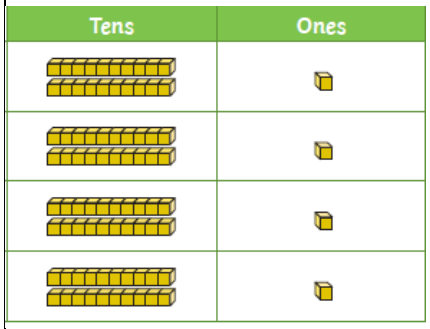
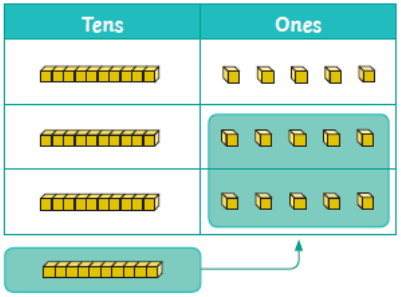
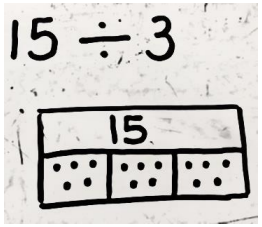
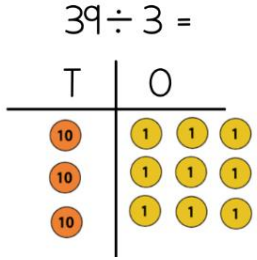
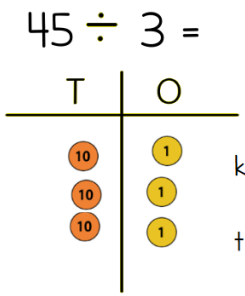
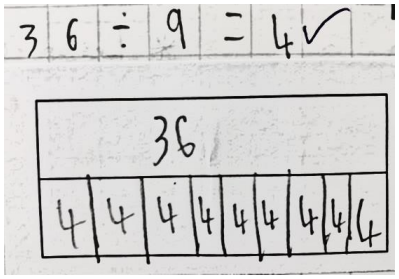
Year 3 – Subtraction			
Objective	Concrete	Pictorial	Abstract
Subtract numbers mentally, including: • a three-digit number and 1s • a three-digit number and 10s • a three-digit number and 100s			Children can use the representations from the concrete and pictorial learning to visualise numbers in their head.

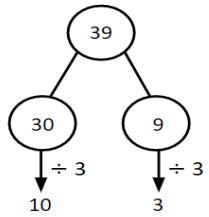
Subtract numbers with up to 3 digits, using formal written methods of columnar subtraction	When starting learning to subtract using Dienes or if confident, a place value grid. Children make the first number and subtract the second where they physically take away the resources (or in this case, cross them out). Initially, children do not cross the tens but then move on to crossing tens where children need to 'Swap Shop' (swapping hundreds for tens or tens for ones)   	   	 

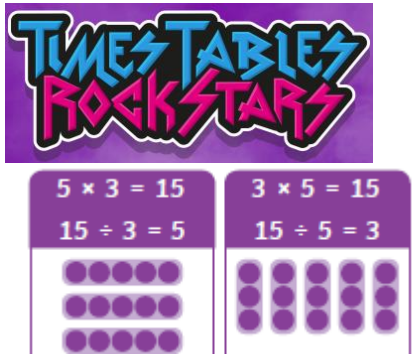
Year 3 – Multiplication			
Objective	Concrete	Pictorial	Abstract

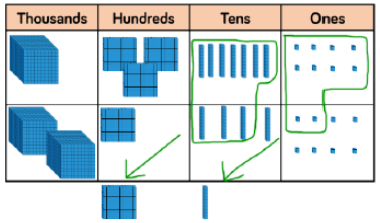
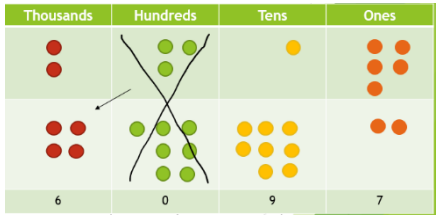
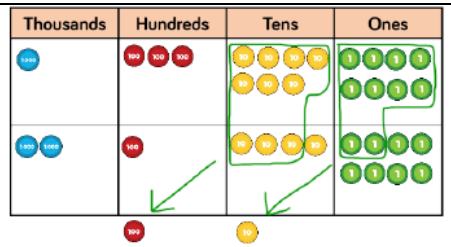
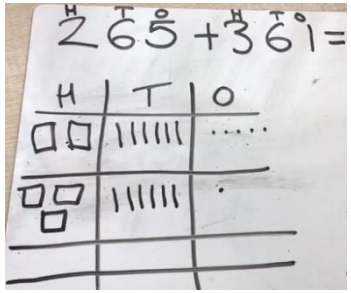
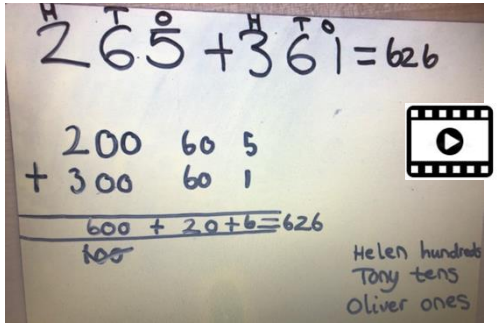
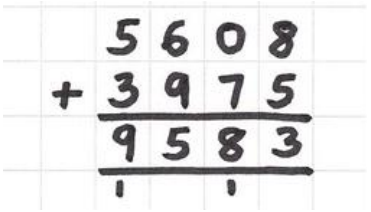
Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods	 Children partition the number they are multiplying into tens and ones and write them on the side of the grid. The number they are multiplying it by goes on the top (we say "tiny on the top"). Children can count the resources of tens and ones and write the answer in the grid. Next, children add the answers (first adding the ones and then the tens). We encourage children to make sure the digits are lined up (tens and ones) to make it easier when adding (as this links to our column addition method).		 Children move away from the concrete and pictorial representations where they can visually count the tens and ones and move onto to using their number knowledge and times tables.
Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects			add

Year 3 – Division			
Objective	Concrete	Pictorial	Abstract

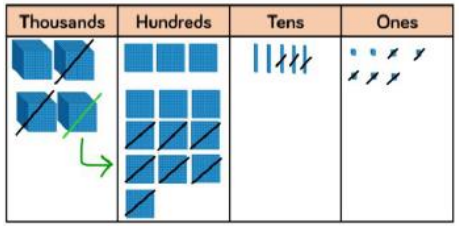
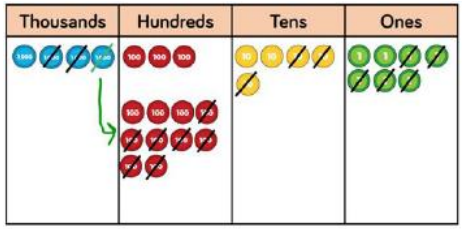
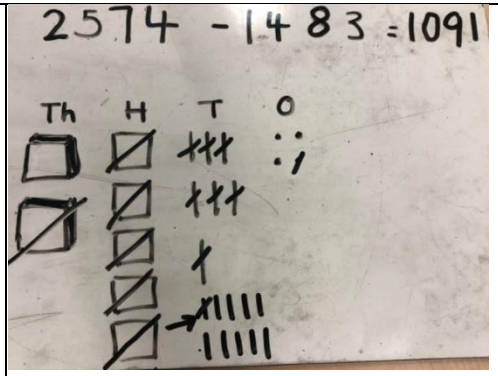
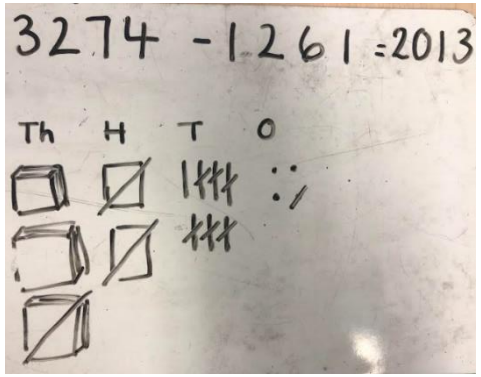
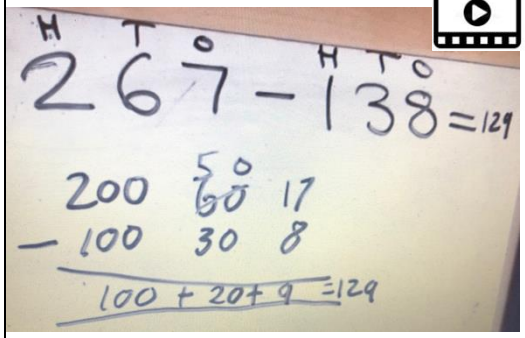
Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods	**Bar model** Children firstly use the bar model method leant in Year 2 to recap their division knowledge. **Dividing 2-digit number ÷ 1-digit number** 84 ÷ 4 – no regrouping required Children partition the first number into tens and ones (80÷4 and 4÷4). Using resources they share the amount equally and then count how much is in each part.  45 ÷ 3 – regrouping required After the initial tens are shared, it is clear that regrouping of the last 10 is required in order to share equally. This 10 is exchanged for ones so that these can be shared. 	**Bar model** The bar method is used as a pictorial representation for sharing. Children draw a bar model and share the dots between each group.  **Dividing 2-digit number ÷ 1-digit number** No regrouping required  Regrouping required  	**Bar model** Children can an abstract representation of the bar model using known multiplication and division facts. **Dividing 2-digit number ÷ 1-digit number**  If children are confident with the previous steps and the concrete / pictorial representations, they can start to use the part whole method. This requires children to use their multiplication and division knowledge to divide mentally.

			39 ÷ 3 = 13  10 + 3 = 13
Solve problems, including missing number problems, involving division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	Need to check		

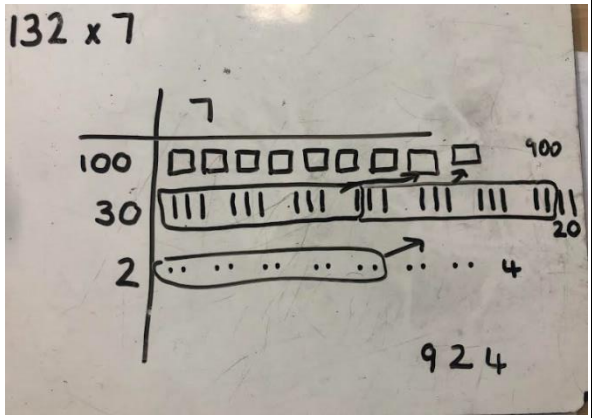
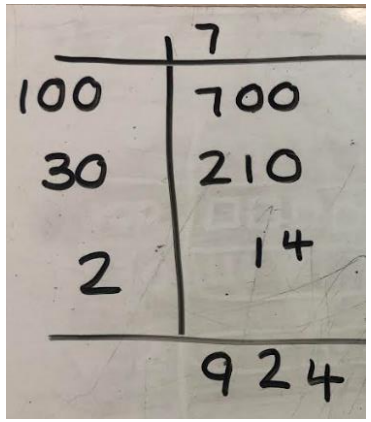
Combined objectives for addition and subtraction			
Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	Songs on YouTube Times table booklets Games Lessons on 3,4 and 8's Hit the Button Multiplicand x multiplier = total 3 x 4 = 12 this is the same as 3+3+3+3 = 12. Learning the correct way to represent a calculation helps them when learning division. However, children learn that multiplication is commutative so they can use other multiplication facts to help them work out a calculation 3 x 5 (they may decide to use their 5 times table rather than their 3 times table).	Add	

Year 4 – Addition			
Objective	Concrete	Pictorial	Abstract
Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate	  Add together the ones first, then tens, hundreds, thousands etc... Use Dienes or if confident, use place value counters. Children do not cross the tens initially but then start to exchange (swapping ones for tens, tens for hundreds etc...) when there is more than 9 in a single column.	  Children move on to using place value counters and / or drawing Dienes to represent addition. The method remains the same as the concrete method.	 Children initially revisit expanded column addition before moving on to contracted column.  For both methods, children start adding from the right. No column can have more than 9 in it so when there is more than 9 so an extra (ten, hundred or thousand) is added to the column on the left.
			Mental methods – bridging

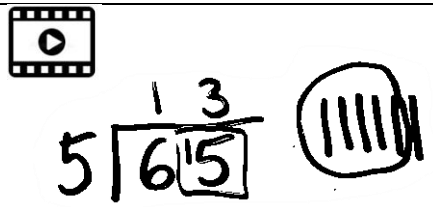
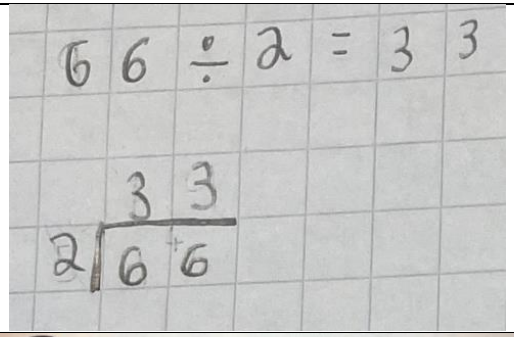
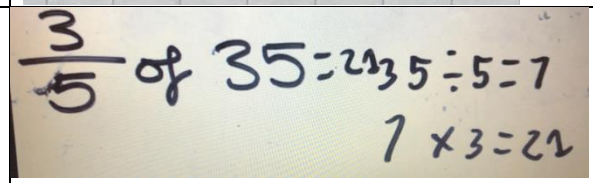
Year 4 – Subtraction			
Objective	Concrete	Pictorial	Abstract

Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate	When start learning to subtract using Dienes or if confident, a place value grid. Children make the first number and subtract the second where they physically take away the resources (or in this case, cross them out). Initially, children do not cross the tens but then move on to crossing tens where children need to 'Swap Shop' (swapping thousands for hundreds, hundreds for tens or tens for ones)   	  Some children find drawing the pictorial method quite complicated because of the thousand blocks. However, it is important that children see the pictorial representation in order to understand the concept. For these children, teachers model this and ask children to explain the steps before moving on to abstract column.	 Children initially revisit expanded column subtraction before moving on to contracted column. 3 1 4357 − 2735 1622 3 1 435 − 273 262 For both methods, children start subtracting from the right (subtraction the bottom number from the top number). If the top number is larger, children exchange from the column to the left in order to create a 2-digit number to subtract from.

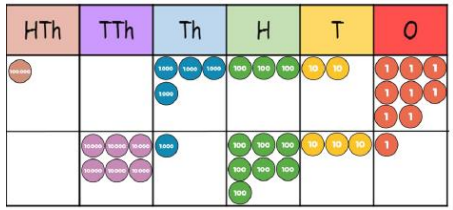
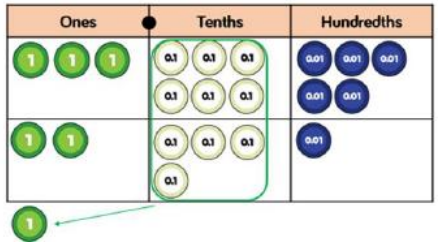
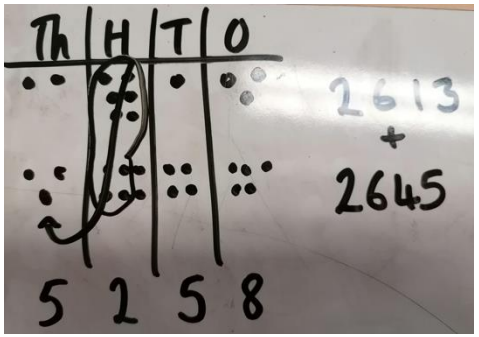
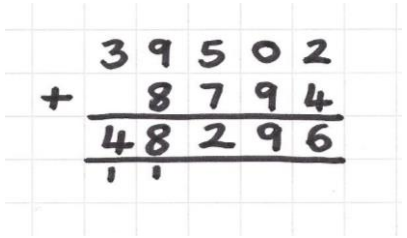
Year 4 – Multiplication

Objective	Concrete	Pictorial	Abstract
Recall multiplication and division facts for multiplication tables up to 12 × 12	Method for teaching these?		
Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1 and multiplying together 3 numbers			
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout			
Solve problems involving multiplying and adding, including			

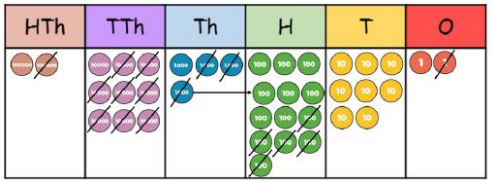
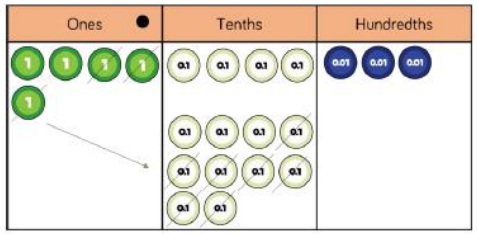
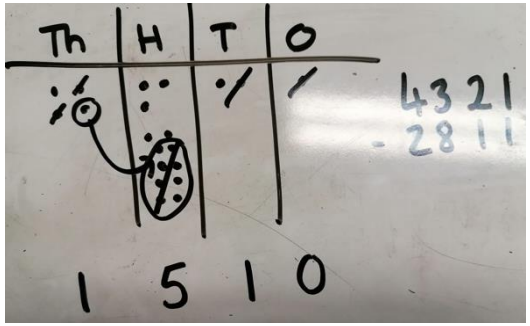
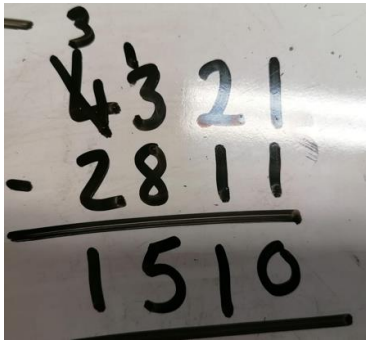
using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects			

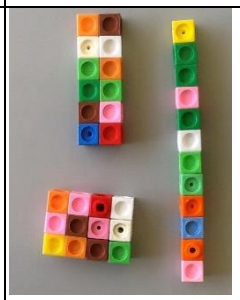
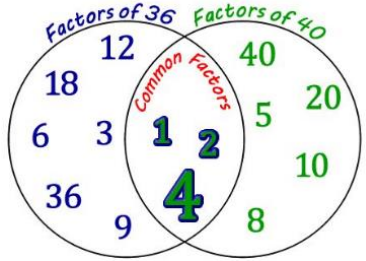
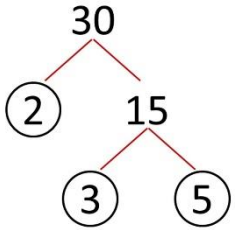
Year 4 – Division			
Objective	Concrete	Pictorial	Abstract
.....			
Recall division facts for multiplication tables up to 12 × 12	600 ÷ 3 = 200 can be derived from 2 x 3 = 6).		 Divide the denominator, multiply the numerator
Use place value, known and derived facts to divide			

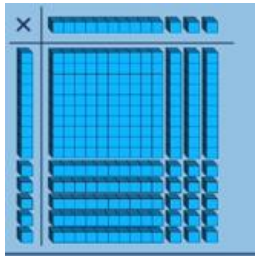
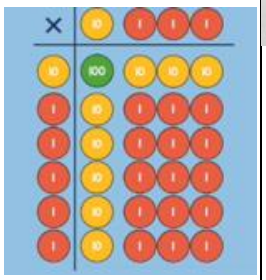
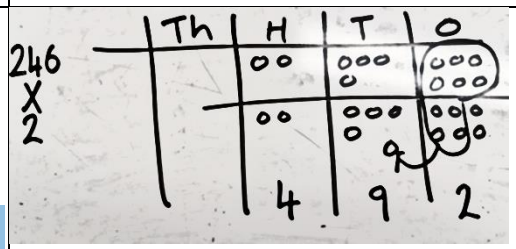
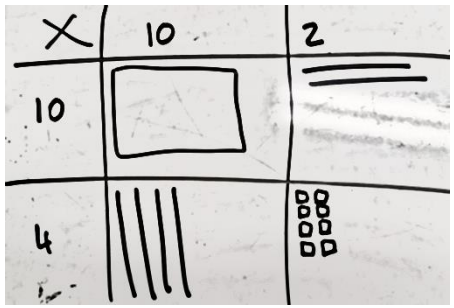
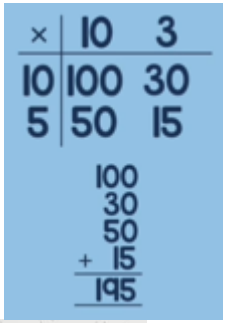
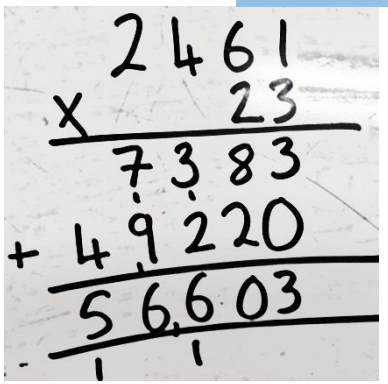
mentally, including: dividing by 1			

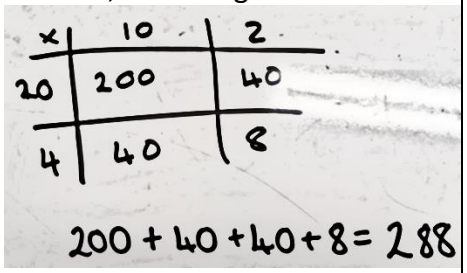
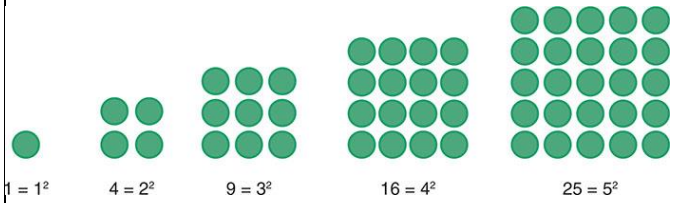
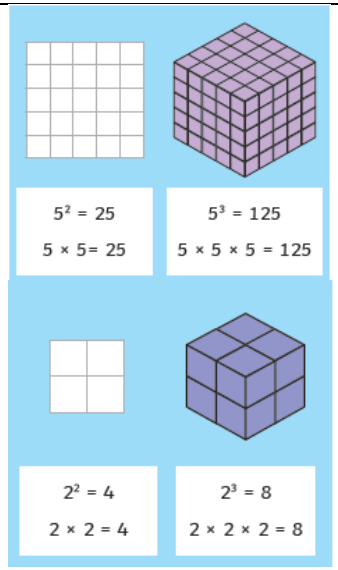
Year 5 – Addition			
Objective	Concrete	Pictorial	Abstract
Add whole numbers with more than 4 digits, including using formal written methods (columnar addition) Add numbers mentally with increasingly large numbers	  Add together the ones first, then tens, hundreds, thousands etc... Use Dienes or if confident, use place value counters. Children do not cross the tens initially but then start to regroup (swapping ones for tens, tens for hundreds etc...)	 Add together the ones first, then tens, hundreds, thousands etc... Children do not cross the tens initially but then start to regroup (swapping ones for tens, tens for hundreds etc...) demonstrating this by circling 10 dots and crossing them out and replacing with one dot in the next power of 10s column.	 Starting with the ones, add each column in turn. Regroup tens, hundreds, thousands and/or tens thousands as required.
Use rounding to check answers to calculations and determine, in the context of a problem,	Rounding is not as accurate when both numbers are rounded up. A better estimate comes from 'rounding' one down and one up.	**Estimating on a number line** **View numbers on a number line to visually**  **determine if they are near the next power of ten and support how accurate the estimate will be.**	41 635 + 7386 = 49 021 Round to ten: 41 630 + 7380 = 49 010 41 630 + 7390 = 49 020 41 640 + 7390 = 49 030

levels of accuracy			

Year 5 – Subtraction			
Objective	Concrete	Pictorial	Abstract
Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction) Subtract numbers mentally with increasingly large numbers	  Start learning to subtract using Dienes or if confident, a place value grid. Children make the first number and then subtract the second where they physically take away the resources (or in this case, cross them out). Initially, children do not cross the tens but then move on to crossing tens where children need to exchange (for example, swapping thousands for hundreds)	 Draw the initial number out in a place value table and starting from the ones subtract the correct given number. When needed cross out the next highest power of ten and replace it with 10 dots in the current column.	5 digit - 5 digit 5 13 1 6 9 7 - 2 6 8 5 4 3 7 8 4 3  Starting with the ones, subtract each column in turn. Exchange tens, hundreds and thousands/ or ten thousands as required.

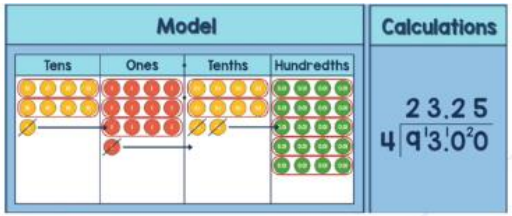
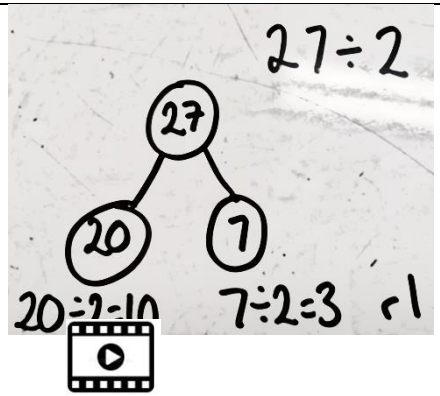
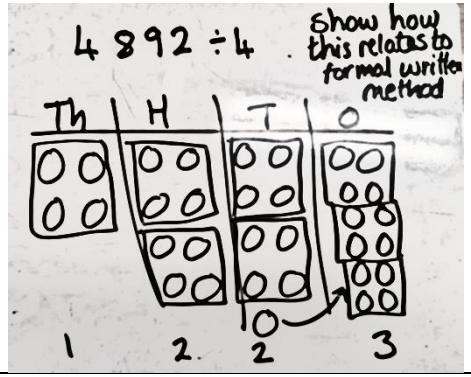
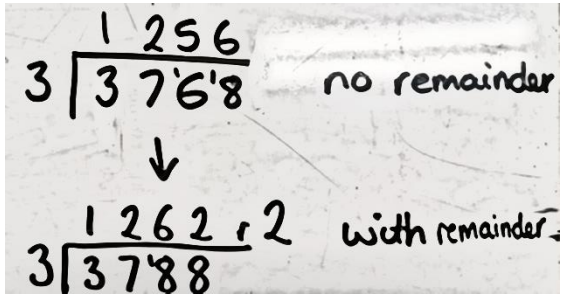
Year 5 – Multiplication			
Objective	Concrete	Pictorial	Abstract
Identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers	 Children use cubes in equal rows to identify factors (how many rows, how many in each row). They use concrete resources to see the relationship of factors and multiples.	Children use arrays to check factors	Children can represent their learning of multiples and factors using a Venn diagram.  Misconception -
Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers Establish whether a		Look at prime numbers on a 100 square and use arrays to show they only have 2 factors.	

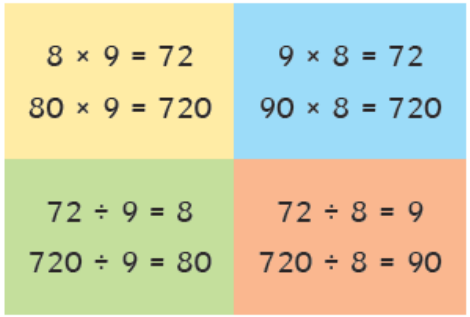
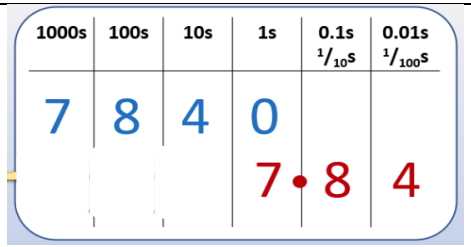
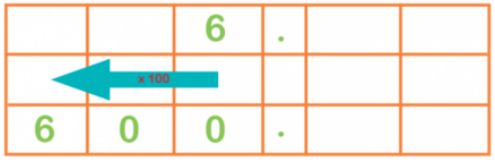
number up to 100 is prime and recall prime numbers up to 19			
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	13 x15   This method is suitable for some children, but confuses others that have become confident using the grid method for multiplication. It is useful if the children don't have a deeper understanding to work through with them in small groups.	  This method is suitable for some children, but confuses others that have become confident using the grid method for multiplication. It is useful if the children don't have a deeper understanding to work through with them in small groups.	  Multiply everything by the ones and place in the top row. Place a place holder in the ones column for the second row and then

			multiply as though it were a 2. Finally add the numbers together. A common mistake is digit reversal, recording 43 and 34 
Solve problems involving multiplication including using their knowledge of factors and multiples, squares and cubes Recognise and use	Use cubes to create arrays and build larger cubes to see cube numbers up to 125 (working together in groups).  	 	

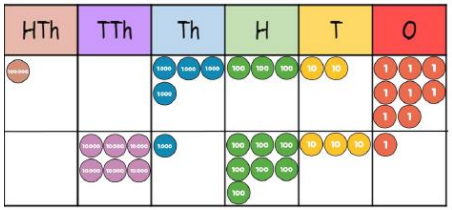
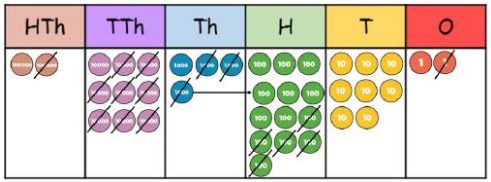
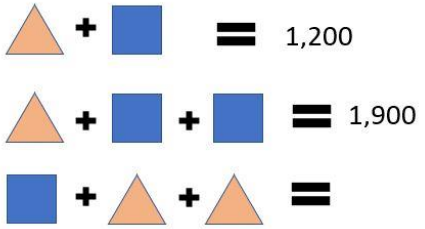
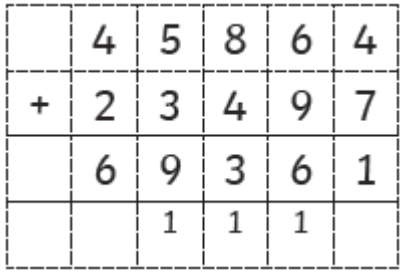
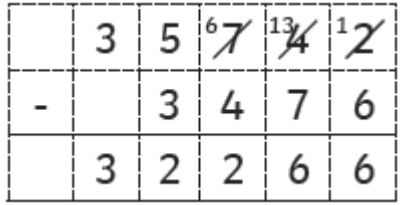
square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})		Draw square numbers out on squared paper and count the squares.	

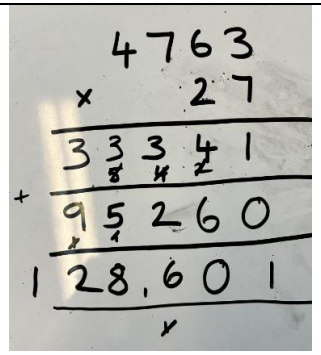
Year 5 – Division			
Objective	Concrete	Pictorial	Abstract

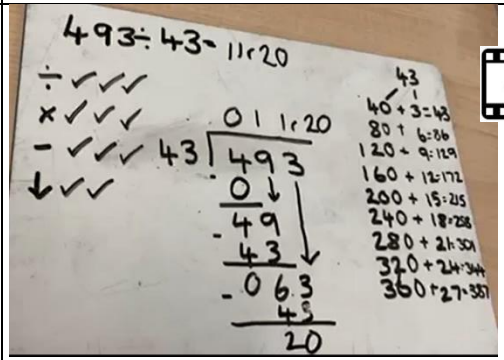
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	 Hasn't worked – can we think how we could simplify this so that they still have a concrete learning opportunity? Children that still need concrete support for division can use cubes to divide a 2 digit number by a 1 digit number. Some children are confused by the method above and this is best for small group work to support their understanding if they are not yet accessing the pictorial.	 	
Solve problems involving division, including using their knowledge of factors and multiples,		Use partitioning to demonstrate how numbers can be broken down. This also support mental calculations. Move on to using a place value table, grouping each column. Remind children that in division (formal written method) we start from the highest value digit as opposed to	interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = \frac{98}{4} = 24$ r 2 = $24\frac{1}{2} = 24.5 \approx 25$

squares and cubes		the smallest value digit in the other 3 operations)	
Combined objectives for addition, subtraction, multiplication and division			
Multiply and divide numbers mentally, drawing upon known facts	For children that still need support they can use Dienes to investigate how the answers to 3x1 compares to the answer to 3x10.		
Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000	Use digit cards on a place value table and physically move them	  Draw place value tables and move the digits left or right depending on the operation and the power of ten	 Think about the number of zeros in a number to mentally move it left/right on a place value table.

		Common misconception is that we can add or take away zeros. This doesn't work when working with numbers that may contain decimals	

Year 6 – Addition and Subtraction			
Objective	Concrete	Pictorial	Abstract
Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	 Add together the ones first, then tens, hundreds, thousands etc... Use Dienes or if confident, use place value counters. Children do not cross the tens initially but then start to regroup (swapping ones for tens, tens for hundreds etc...)  Start learning to subtract using Dienes or if confident, a place value grid. Children make the first number and then subtract the second where they physically take away the resources (or in this case, cross them out). Initially, children do not cross the tens but then move on to crossing tens where children need to exchange (for example, swapping thousands for hundreds)	 Using missing number questions, so that children have to explore whether they need to use addition or subtraction or a combination of both.	 Starting with the ones, add each column in turn. Regroup tens, hundreds, thousands, ten thousands as required.  Starting with the ones, subtract each column in turn. Exchange tens, hundreds, thousands and/or ten thousands as required.

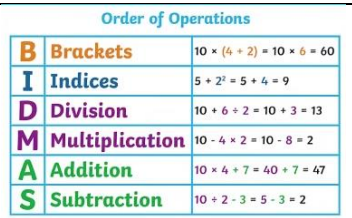
Year 6 – Multiplication			
Objective	Concrete	Pictorial	Abstract
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication	Building on from Year 5, children use concrete resources to show their initial understanding of multiplication. This knowledge is then applied to working with larger numbers until 4 digit numbers are multiplied by 2 digit numbers using short multiplication.		

Year 6 – Division			
Objective	Concrete	Pictorial	Abstract
Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as	Bus stop is revisited and taught using the same methods as in Year 5. No concrete / pictorial representation is used for longer division.		 Children partition the second number into tens and ones. They then list 9 lots of the tens and 9 lots of the ones (this speeds up the method).

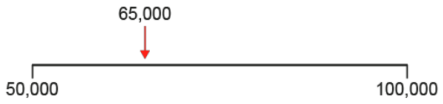
<table>
<tr><td>appropriate for the context

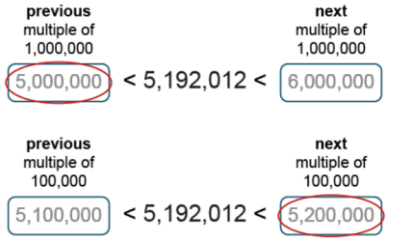
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context</td><td></td><td>Children then write the steps of the method.
÷ - Daddy divide
X - Mummy multiply
- Sister subtract
↓ Brother - break down

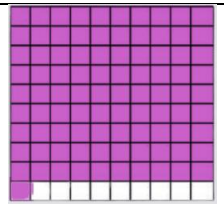
The number they are dividing goes into the bus stop and the number they are dividing it by goes on to the right.

Children then follow the steps (÷ x – and ↓) start with looking at the first digit in the bus stop.</td></tr>
<tr><td colspan="3">Combined objectives for the four operations</td></tr>
<tr><td>Use their knowledge of the order of operations to carry out calculations involving the 4 operations</td><td>BIDMAS is taught once the children have recapped all 4 operations using concrete, pictorial and abstract representations. No concrete / pictorial representation is used for carrying out calculations using all four operations.</td><td> 

36 ÷ (7 - 3) =

Children identify which part of the calculation they need to first and work out the answer. They then move on to the next part of the calculation according to the rules of BIDMAS until they have completed the whole calculation.</td></tr>
<tr><td>Use estimation to check answers to calculations and determine, in the</td><td>Rounding is not as accurate when both numbers are rounded up. A better estimate</td><td> 
</td></tr>
</table>

context of a problem, an appropriate degree of accuracy	comes from 'rounding' one down and one up.	Children need to be able to estimate the placing of numbers on an unmarked number line using appropriate proportional reasoning.
Round answers to a specified degree of accuracy, for example, to the nearest 10,000, 100,000 and 1,000,000 etc, but not to a specified number of significant figures	No concrete / pictorial representation is used for rounding.	 When rounding, children use the language of: "The previous multiple of _____ is. The next multiple of _____ is."
Solve problems involving ratio relationships	 Children recognise a 1-to-many or many-to-1 structure and use the relationship to solve problems. For example, here children should recognise that, in both examples, for every 1 red bead there are 3 blue beads (or for every 3 blue beads there is 1 red bead), irrespective of the arrangement of the beads.	Using the pictorial representation, children answer questions such as: If there were 5 red beads, how many blue beads would there be? If there were 21 blue beads, how many beads would there be altogether? If there were 40 beads altogether, how many red beads and how many blue beads would there be?

Solve problems involving the calculations of percentages (including within measures) and use percentages for comparison.	 Using Dienes to represent one whole and then a percentage of that whole.	Use the bar model to help you. 120 kg 50% 50% 25% 25% 25% 25% What is 50% of 120 kg? What is 25% of 120 kg?	Max has £800 in the bank. He spends 3% of his money on a new computer game. How much money does he spend on the computer game? £ Children use their knowledge of finding a percentage of a quantity to solving measure problems.